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An optimal control algorithm for entrance concurrent flow problems

Cheng-Hung Huang *, Chun-Ying Yeh

Department of Naval Architecture and Marine Engineering, National Cheng Kung University, Tainan 701, Taiwan, ROC

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Abstract

The conjugate gradient method of minimization is applied successfully in the present optimal control algorithm in determining the optimal boundary control function for a concurrent flow problem based on the desired thermal entry length and fluid temperatures.

The validity of the present optimal control analysis is examined by means of numerical experiments. Different desired thermal entry length and fluid temperature distributions are given in three test cases and the corresponding optimal control heat fluxes are determined. The results show that the optimal boundary heat fluxes can be obtained with an arbitrary initial guess within seconds of CPU time on a Pentium III-600 MHz PC.

1. Introduction

The problem of the simultaneous determination of both fluid temperatures for entrance concurrent flow concerns all usual double pipe, concurrent flow heat exchangers consisting of two parallel plate channels or concentric circular tubes when fluids at different temperatures enter the annular space and central tube at the same side of the heat exchanger.

The traditional methods of predicting heat transfer in such situations are based on the assignment of heat transfer coefficients for each flow regardless of the actual coupling of the boundary condition. Such a coupling can be important in the thermal entry regions, especially with laminar flow and therefore the problem of heat transfer at the entrance region in concurrent flow double pipe heat exchanger has been carefully investigated [1–3].

Sometimes, it may be of important to design the desired length of thermal entry region or to design the desired fluid temperatures in the double pipe, concurrent flow heat exchanger for some specific applications or experiments by applying the optimal boundary control functions. Under this consideration, an optimal control algorithm should be combined with the concurrent flow problem to match the requirement.

Optimal control laws have received increased attention in recent years in many engineering applications. Such system can be controlled either at the boundary (boundary control) or through the spatial domain (distributed control), or both.

Optimal control techniques have been applied in many different area of research, especially in heat transfer engineering. This type of problems has been initiated by Butkovskii and Lerner [4]. Meric [5,6] used the conjugate gradient method (CGM) to find the optimal boundary control temperatures for a non-linear system, i.e. temperature-dependent thermal properties. Chen and Ozisik used similar algorithm to determine the optimal heating sources for a slab [7,8] and for a cylinder [9] in a non-linear optimal control problem. Recently, Huang used a similar algorithm to determine the optimal boundary control functions for a non-linear heat transfer problem [10] and to estimate the unknown control

* Corresponding author. Tel.: +886-6-274-7018; fax: +886-6-274-7019.
E-mail address: chhuang@mail.ncku.edu.tw (C.-H. Huang).

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Nomenclature

H heat capacity flow rate ratio
J functional defined by Eq. (2)
J' gradient of functional defined by Eq. (15)
K fluid thermal resistance ratio
K_w wall thermal resistance ratio
P direction of descent defined by Eq. (4)
$q(z)$ control function
$U_1(R)$, $U_2(R)$ velocity profiles
Y_1, Y_2 desired fluid temperatures

Greek symbols
β search step size
γ conjugate coefficient
θ_1, θ_2 dimensionless fluid temperatures
$\Delta\theta_1$, $\Delta\theta_2$ sensitivity functions defined by Eqs. (6a)–(6d)
λ_1, λ_2 Lagrange multipliers defined by Eqs. (12a)–(12d)

Superscript
n iteration index

forces for a forced vibration problem [11]. The optimal control problems considered above are not for the coupled system.

The objective of the present study is to solve the coupled optimal control problem for concurrent flow in determining the boundary control function based on the desired thermal entry length and fluid temperatures by using the CGM. Moreover, an explicit expression for the determination of search step sizes is also derived with the help of the solutions of sensitivity problem.

The CGM of optimal control derives basis from the perturbation theory [12] and transforms an inverse problem to the solution of three problems, namely, the direct problem, a sensitivity problem and an adjoint problem, which will be discussed in detail in the following text.

2. Direct problem

To illustrate the methodology for developing expressions for use in determining the optimal boundary control heat flux for concurrent flow problem, we consider the following optimal control problem. Two fluids at different temperatures enter through the same inlet of the parallel plane channels. The upper boundary surface is subjected to a control heat flux $q(z)$. The geometry and coordinates for this concurrent flow problem is illustrated in Fig. 1.

The dimensionless formulation of the coupled concurrent flow problem can be expressed as [1]:

$$\begin{cases} U_1(R)\dfrac{\partial\theta_1(R,z)}{\partial z} = \dfrac{\partial^2\theta_1(R,z)}{\partial R^2} \\ KHU_2(R)\dfrac{\partial\theta_2(R,z)}{\partial z} = \dfrac{\partial^2\theta_2(R,z)}{\partial R^2} \end{cases} \quad \text{in } 0 < R < 1,\ \ 0 < z < z_f \tag{1a}$$

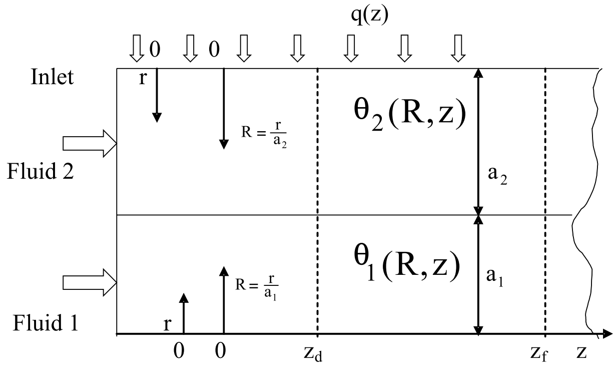


Fig. 1. Geometry and coordinates for concurrent flow double pipe heat exchangers.

$$\begin{cases} \dfrac{\partial\theta_1(0,z)}{\partial R} = 0 \\ \dfrac{\partial\theta_2(0,z)}{\partial R} = q(z) \end{cases} \quad \text{at } R = 0 \tag{1b}$$

$$\begin{cases} K\dfrac{\partial\theta_1(1,z)}{\partial R} + \dfrac{\partial\theta_2(1,z)}{\partial R} = 0 \\ K_{\mathrm{w}}\dfrac{\partial\theta_1(1,z)}{\partial R} + \theta_1(1,z) - \theta_2(1,z) = 0 \end{cases} \quad \text{at } R = 1 \tag{1c}$$

$$\begin{cases} \theta_1(R,0) = 0 \\ \theta_2(R,0) = 1 \end{cases} \quad \text{at } z = 0 \tag{1d}$$

Here θ_1 and θ_2 are the temperatures in the central (fluid 1) and annular (fluid 2) space, respectively. The coordinate R for the inner tube is measured from the center of the tube, and the coordinate R for the outer tube is measured from the outer wall of the tube. K, H and K_{w} are dimensionless numbers defined in [1] and called fluid thermal resistance ratio, heat capacity flow rate ratio and wall thermal resistance ratio, respectively. For the laminar flow case $U_1(R) = 2(1 - R^2)$ and $U_2(R) = 6R(1 - R)$.

It is obvious from Eq. (1c) that two fluids are coupled at the boundary $R = 1$. The direct problem considered here is concerned with calculating the fluid temperature when the control function $q(z)$, thermal properties and inlet conditions are known. The implicit finite-difference method can be used to solve this direct problem.

3. Optimal control problem

In the optimal control problem, the control function $q(z)$ is regarded as being unknown, but everything else in Eqs. (1a)–(1d) is known. In addition, the desired thermal entry length and fluid temperature distributions within the specified domain (i.e. from z_{d} to z_{f}) are considered available.

Let the desired fluid temperatures be denoted by $Y_1(R,z)$ and $Y_2(R,z)$ with desired thermal entry length z_{d}. Then this optimal control problem can be stated as follows: by utilizing the above mentioned desired thermal entry length z_{d} and temperature data $Y_1(R,z)$ and $Y_2(R,z)$, estimate the strength of the optimal control function $q(z)$ over the specified boundary to match this requirement.

The solution of the present optimal control problem is to be obtained in such a way that the following functional is minimized:

$$J[q(z)] = \int_{z=z_{\mathrm{d}}}^{z_{\mathrm{f}}} \int_{R=0}^{1} \{[\theta_1(R,z) - Y_1(R,z)]^2 + [\theta_2(R,z) - Y_2(R,z)]^2\}\,\mathrm{d}R\,\mathrm{d}z + \frac{\alpha}{2}\int_{z=0}^{z_{\mathrm{f}}} q(z)^2\,\mathrm{d}z \tag{2}$$

Here α is a given weighting coefficient. $\theta_1(R,z)$ and $\theta_2(R,z)$ are the estimated or computed temperatures within the specified domain. These quantities are determined from the solution of the direct problem given previously by using the estimated control function $q(z)$.

The first term on the right hand side is the integration of the square of the deviation between the estimated and desired temperatures for two different fluids. The second term is the integration with respect to z of the square of the control function $q(z)$, over the control surface z_{f} multiplied by the weighting coefficient α.

The square of the control function (i.e. the quadratic form) guarantees the existence of the minimum and avoid the cancellation effect between the positive and negative values.

The weighting coefficient α is the design parameter that control the closeness of the estimated temperatures to the desired temperatures. It can also be used as the adjustment factor of the control function $q(z)$. When there exist some reasons such that the control function cannot be applied as what we have calculated, under this circumstance we should increase the value of weighting coefficient. As a result, the estimated control function will be damped and the supplying rate of heat fluxes can be satisfied.

4. Conjugate gradient method for minimization

The following iterative process based on the CGM [12] is now used for the estimation of control function $q(z)$ by minimizing the above functional $J[q(z)]$:

$$q^{n+1}(z) = q^n(z) - \beta^n P^n(z), \quad n = 0, 1, 2, \ldots \tag{3}$$

where β^n is the search step size in going from iteration n to iteration $n+1$, and $P^n(z)$ is the direction of descent (i.e. search direction) given by

$$P^n(z) = J'^n(z) + \gamma^n P^{n-1}(z) \tag{4}$$

which is a conjugation of the gradient direction $J'^n(z)$ at iteration n and the direction of descent $P^{n-1}(z)$ at iteration $n-1$. The conjugate coefficient is determined from

$$\gamma^n = \frac{\int_{z=0}^{z_f} [J'^n(z)]^2 \, dz}{\int_{z=0}^{z_f} [J'^{n-1}(z)]^2 \, dz} \quad \text{with } \gamma^0 = 0 \tag{5}$$

To perform the iterations according to Eq. (4), we need to compute the step size β^n and the gradient of the functional $J'^n(z)$. In order to develop expressions for the determination of these two quantities, a "sensitivity problem" and an "adjoint problem" are constructed as described below.

5. Sensitivity problem and search step size

It is assumed that when the control function $q(z)$ undergoes a variation $\Delta q(z)$, $\theta_1(R,z)$ and $\theta_2(R,z)$ are perturbed by $\Delta\theta_1(R,z)$ and $\Delta\theta_2(R,z)$, respectively. Then replacing in the direct problem $q(z)$ by $q(z) + \Delta q(z)$, $\theta_1(R,z)$ by $\theta_1(R,z) + \Delta\theta_1(R,z)$, and $\theta_2(R,z)$ by $\theta_2(R,z) + \Delta\theta_2(R,z)$, subtracting from the resulting expressions the direct problem and neglecting the second-order terms, the following sensitivity problem for the sensitivity functions $\Delta\theta_1(R,z)$ and $\Delta\theta_2(R,z)$ are obtained.

$$\begin{cases} U_1(R)\dfrac{\partial \Delta\theta_1(R,z)}{\partial z} = \dfrac{\partial^2 \Delta\theta_1(R,z)}{\partial R^2} \\ KHU_2(R)\dfrac{\partial \Delta\theta_2(R,z)}{\partial z} = \dfrac{\partial^2 \Delta\theta_2(R,z)}{\partial R^2} \end{cases} \quad \text{in } 0 < R < 1, \ 0 < z < z_f \tag{6a}$$

$$\begin{cases} \dfrac{\partial \Delta\theta_1(0,z)}{\partial R} = 0 \\ \dfrac{\partial \Delta\theta_2(0,z)}{\partial R} = \Delta q(z) \end{cases} \quad \text{at } R = 0 \tag{6b}$$

$$\begin{cases} K\dfrac{\partial \Delta\theta_1(1,z)}{\partial R} + \dfrac{\partial \Delta\theta_2(1,z)}{\partial R} = 0 \\ K_w\dfrac{\partial \Delta\theta_1(1,z)}{\partial R} + \Delta\theta_1(1,z) - \Delta\theta_2(1,z) = 0 \end{cases} \quad \text{at } R = 1 \tag{6c}$$

$$\begin{cases} \Delta\theta_1(R,0) = 0 \\ \Delta\theta_2(R,0) = 0 \end{cases} \quad \text{at } z = 0 \tag{6d}$$

We should note that the above sensitivity problems can be solved by using the implicit finite-difference method.
The functional $J(q^{n+1})$ for iteration $n+1$ is obtained by rewriting Eq. (2) as

$$J[q^{n+1}(z)] = \int_{z=z_d}^{z_f} \int_{R=0}^{1} [\theta_1(R,z;q^n - \beta^n P^n) - Y_1(R,z)]^2 \, dR \, dz + \int_{z=z_d}^{z_f} \int_{R=0}^{1} [\theta_2(R,z;q^n - \beta^n P^n) - Y_2(R,z)]^2 \, dR \, dz + \frac{\alpha}{2}\int_{z=0}^{z_f} (q^n - \beta^n P^n)^2 \, dz \tag{7}$$

where we replaced $q^{n+1}(z)$ by the expression given by Eq. (3). If the estimated temperatures $\theta_1(R,z;q^n - \beta^n P^n)$ and $\theta_2(R,z;q^n - \beta^n P^n)$ are linearized by a Taylor expansion, Eq. (7) takes the form

$$J[q^{n+1}(z)] = \int_{z=z_d}^{z_f} \int_{R=0}^{1} [\theta_1(R,z;q^n) - \beta^n \Delta\theta_1(P^n) - Y_1(R,z)]^2 \, dR \, dz + \int_{z=z_d}^{z_f} \int_{R=0}^{1} [\theta_2(R,z;q^n) - \beta^n \Delta\theta_2(P^n) - Y_2(R,z)]^2 \, dR \, dz + \frac{\alpha}{2}\int_{z=0}^{z_f} (q^n - \beta^n P^n)^2 \, dz \tag{8}$$

where $\theta_1(R,z;q^n)$ and $\theta_2(R,z;q^n)$ are the solutions of the direct problem by using estimate control function $q(z)$.

The sensitivity functions $\Delta\theta_1(P^n)$ and $\Delta\theta_2(P^n)$ are taken as the solutions of problems (6a)–(6d) and (7) by letting $\Delta q(z) = P^n(z)$.

Eq. (8) is differentiated with respect to β^n, and equating it equal to zero to obtain the following expression for the search step size β^n as

$$\beta^n = \frac{\int_{z=z_d}^{z_f}\int_{R=0}^{1}[2(\theta_1 - Y_1)\,\Delta\theta_1(P^n) + 2(\theta_2 - Y_2)\,\Delta\theta_2(P^n)]\,\mathrm{d}R\,\mathrm{d}z + \int_{z=0}^{z_f}\alpha q^n P^n\,\mathrm{d}z}{\int_{z=z_d}^{z_f}\int_{R=0}^{1}[2\,\Delta\theta_1^2(P^n) + 2\,\Delta\theta_2^2(P^n)]\,\mathrm{d}R\,\mathrm{d}z + \int_{z=0}^{z_f}\alpha P^{n2}\,\mathrm{d}z} \tag{9}$$

6. Adjoint problem and gradient equation

To obtain the adjoint problem, Eq. (1a) is multiplied by the Lagrange multiplier (or adjoint function) $\lambda_1(R,z)$ and $\lambda_2(R,z)$ and the resulting expression is integrated over the specified domain. Then the result is added to the right hand side of Eq. (2) to yield the following expression for the functional $J[q(z)]$:

$$J[q(z)] = \int_{z=z_d}^{z_f}\int_{R=0}^{1}\{[\theta_1(R,z) - Y_1(R,z)]^2 + [\theta_2(R,z) - Y_2(R,z)]^2\}\,\mathrm{d}R\,\mathrm{d}z + \frac{\alpha}{2}\int_{z=0}^{z_f} q(z)^2\,\mathrm{d}z + \int_{z=0}^{z_f}\int_{R=0}^{1}\lambda_1\left[\frac{\partial^2\theta_1(R,z)}{\partial R^2} - U_1(R)\frac{\partial\theta_1(R,z)}{\partial z}\right]\mathrm{d}R\,\mathrm{d}z + \int_{z=0}^{z_f}\int_{R=0}^{1}\lambda_2\left[\frac{\partial^2\theta_2(R,z)}{\partial R^2} - KHU_2(R)\frac{\partial\theta_2(R,z)}{\partial z}\right]\mathrm{d}R\,\mathrm{d}z, \quad \text{in } 0 < R < 1,\ 0 < z < z_f \tag{10}$$

Firstly, the variation ΔJ is obtained by perturbing $q(z)$ by $q(z) + \Delta q(z)$, $\theta_1(R,z)$ by $\theta_1(R,z) + \Delta\theta_1(R,z)$, and $\theta_2(R,z)$ by $\theta_2(R,z) + \Delta\theta_2(R,z)$ in Eq. (10), subtracting from the resulting expression the original Eq. (10) and neglecting the second-order terms. We thus find

$$\Delta J[q(z)] = \int_{z=0}^{z_f}\int_{R=0}^{1}\{2[\theta_1 - Y_1]\Delta\theta_1 + 2[\theta_2 - Y_2]\Delta\theta_2\}u(z - z_d)\,\mathrm{d}R\,\mathrm{d}z + \alpha\int_{z=0}^{z_f} q(z)\,\Delta q\,\mathrm{d}z + \int_{z=0}^{z_f}\int_{R=0}^{1}\lambda_1\left[\frac{\partial^2\Delta\theta_1(R,z)}{\partial R^2} - U_1(R)\frac{\partial\Delta\theta_1(R,z)}{\partial z}\right]\mathrm{d}R\,\mathrm{d}z + \int_{z=0}^{z_f}\int_{R=0}^{1}\lambda_2\left[\frac{\partial^2\Delta\theta_2(R,z)}{\partial R^2} - KHU_2(R)\frac{\partial\Delta\theta_2(R,z)}{\partial z}\right]\mathrm{d}R\,\mathrm{d}z, \quad \text{in } 0 < R < 1,\ 0 < z < z_f \tag{11}$$

Here $u(\cdot)$ represents a step function. In Eq. (11), the double integral terms are integrated by parts; the boundary conditions of the sensitivity problem are utilized. The vanishing of the integrands leads to the following adjoint problem for the determination of $\lambda_1(R,z)$ and $\lambda_2(R,z)$:

$$\begin{cases} \dfrac{\partial^2\lambda_1(R,z)}{\partial R^2} + U_1(R)\dfrac{\partial\lambda_1(R,z)}{\partial z} + 2(\theta_1 - Y_1)u(z - z_d) = 0 \\ \dfrac{\partial^2\lambda_2(R,z)}{\partial R^2} + KHU_2(R)\dfrac{\partial\lambda_2(R,z)}{\partial z} + 2(\theta_2 - Y_2)u(z - z_d) = 0 \end{cases} \quad \text{in } 0 < R < 1,\ 0 < z < z_f \tag{12a}$$

$$\begin{cases} \dfrac{\partial\lambda_1(0,z)}{\partial R} = 0 \\ \dfrac{\partial\lambda_2(0,z)}{\partial R} = 0 \end{cases} \quad \text{at } R = 0 \tag{12b}$$

$$\begin{cases} \dfrac{\partial\lambda_1(1,z)}{\partial R} + \dfrac{\partial\lambda_2(1,z)}{\partial R} = 0 \\ K_w\dfrac{\partial\lambda_1(1,z)}{\partial R} + \lambda_1(1,z) - K\lambda_2(1,z) = 0 \end{cases} \quad \text{at } R = 1 \tag{12c}$$

$$\begin{cases} \lambda_1(R,z_f) = 0 \\ \lambda_2(R,z_f) = 0 \end{cases} \quad \text{at } z = z_f \tag{12d}$$

The adjoint problem is different from the direct problem in that the final position conditions at $z = z_f$ is specified instead of the customary initial position condition. However, this problem can be transformed to a standard problem by the transformation of the variable as $Z = z_f - z$. Then the standard techniques of implicit finite differences method can be used to solve the above adjoint problem.

Finally, the following integral term is left

$$\Delta J = \int_{z=0}^{z_f} [\lambda_2(0,z) + \alpha q(z)]\, \Delta q \,\mathrm{d}z \tag{13}$$

From definition [12], the functional increment can be presented as

$$\Delta J = \int_{z=0}^{z_f} (J' \Delta q) \,\mathrm{d}z \tag{14}$$

A comparison of Eqs. (13) and (14) leads to the following expression for the gradient of functional $J'[q(z)]$:

$$J'[q(z)] = \lambda_2(0,z) + \alpha q(z) \tag{15}$$

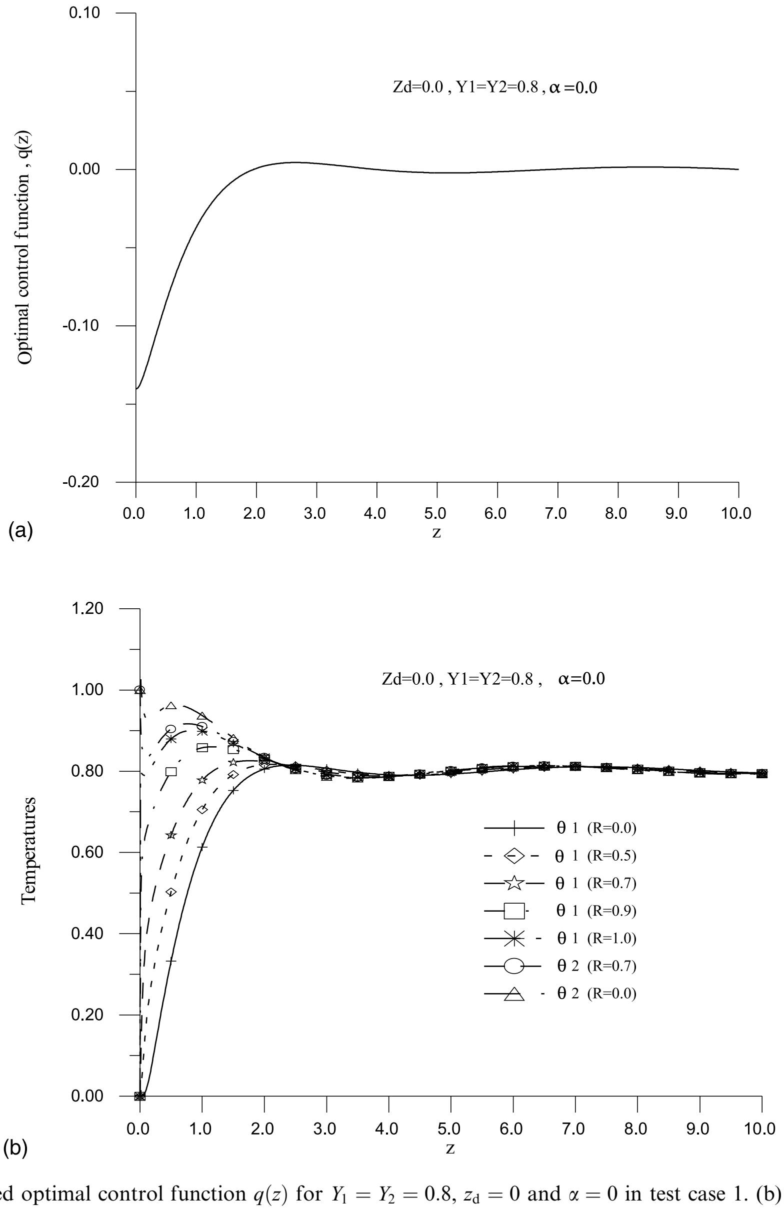


Fig. 2. (a) The estimated optimal control function $q(z)$ for $Y_1 = Y_2 = 0.8$, $z_d = 0$ and $\alpha = 0$ in test case 1. (b) The estimated temperatures, θ_1 and θ_2, for $Y_1 = Y_2 = 0.8$, $z_d = 0$ and $\alpha = 0$ in test case 1.

7. Computational procedure

The computational procedure for the solution of this optimal control problem may be summarized as follows:

Suppose $q^n(z)$ is available at iteration n

Step 1. Solve the direct problem given by Eqs. (1a)–(1d) for $\theta_1(R,z)$ and $\theta_2(R,z)$.

Step 2. Solve the adjoint problem given by Eqs. (12a)–(12d) for $\lambda_1(R,z)$ and $\lambda_2(R,z)$.

Step 3. Compute the gradient of the functional $J'[q(z)]$ from Eq. (15).

Step 4. Compute the conjugate coefficients γ^n and the direction of descent $P^n(z)$ from Eqs. (5) and (4), respectively.

Step 5. Set $\Delta q(z) = P^n(z)$, and solve the sensitivity problems given by Eqs. (6a)–(6d) for $\Delta\theta_1(R,z)$ and $\Delta\theta_2(R,z)$, respectively.

Step 6. Compute the search step size β^n from Eq. (9).

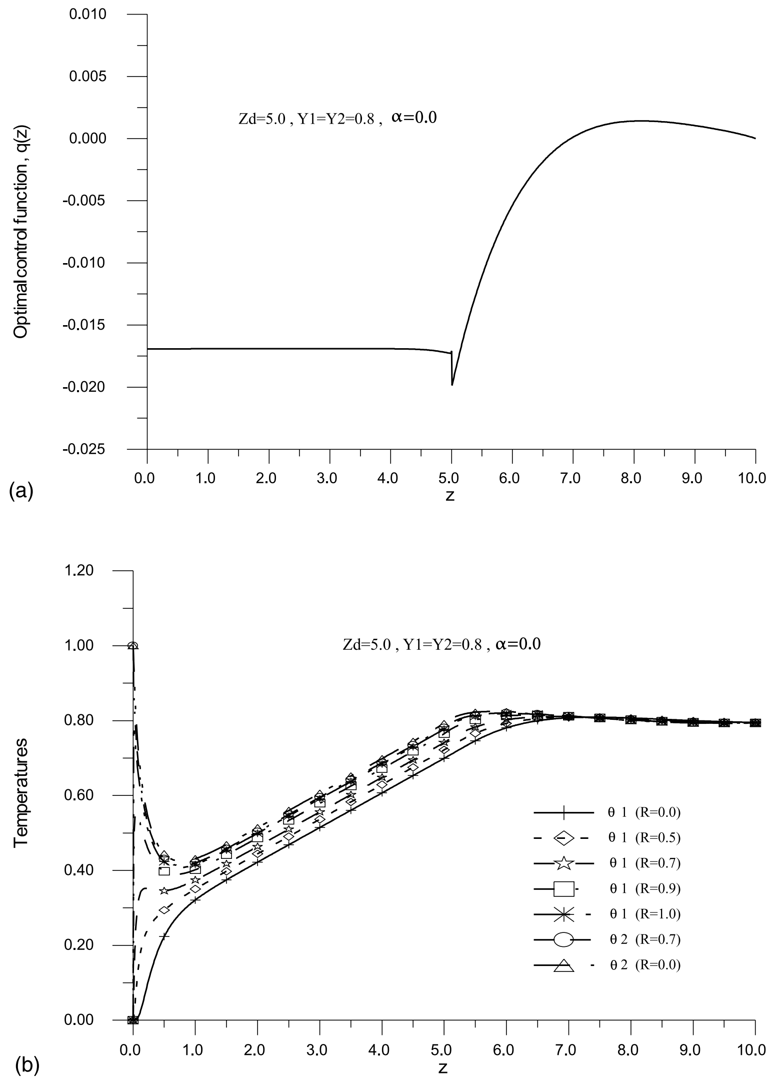


Fig. 3. (a) The estimated optimal control function $q(z)$ for $Y_1 = Y_2 = 0.8$, $z_d = 5$ and $\alpha = 0$ in test case 1. (b) The estimated temperatures, θ_1 and θ_2, for $Y_1 = Y_2 = 0.8$, $z_d = 5$ and $\alpha = 0$ in test case 1.

Step 7. Compute the new estimation for $q^n(z)$ from Eq. (3) and return to Step 1 until the given number of iteration is satisfied.

8. Results and discussions

The physical model for the usual double pipe heat exchanger consists of two parallel plane channels is examined in the present study. Two fluids at different temperatures, $\theta_1(R,0) = 0.0$ and $\theta_2(R,0) = 1.0$, respectively, enter through the same inlet. It is required that the temperature distributions for two fluids be satisfied with the desired uniform temperature for a specified thermal entry length z_d by controlling the strength of boundary heat flux $q(z)$ on the plane surface.

The length of tube is taken as $z_f = 10$ in the present study and the space increments used in numerical calculations are taken as $\Delta R = 0.01$ (i.e. 200 grid points in R space) and $\Delta z = 0.01$ (i.e. 1000 grid points in z space), respectively. The fluid thermal resistance ratio is taken as $K = 0.1$, the heat capacity flow rate ratio is chosen as $H = 0.5$ and the wall thermal resistance ratio is assumed as $K_w = 0.0$.

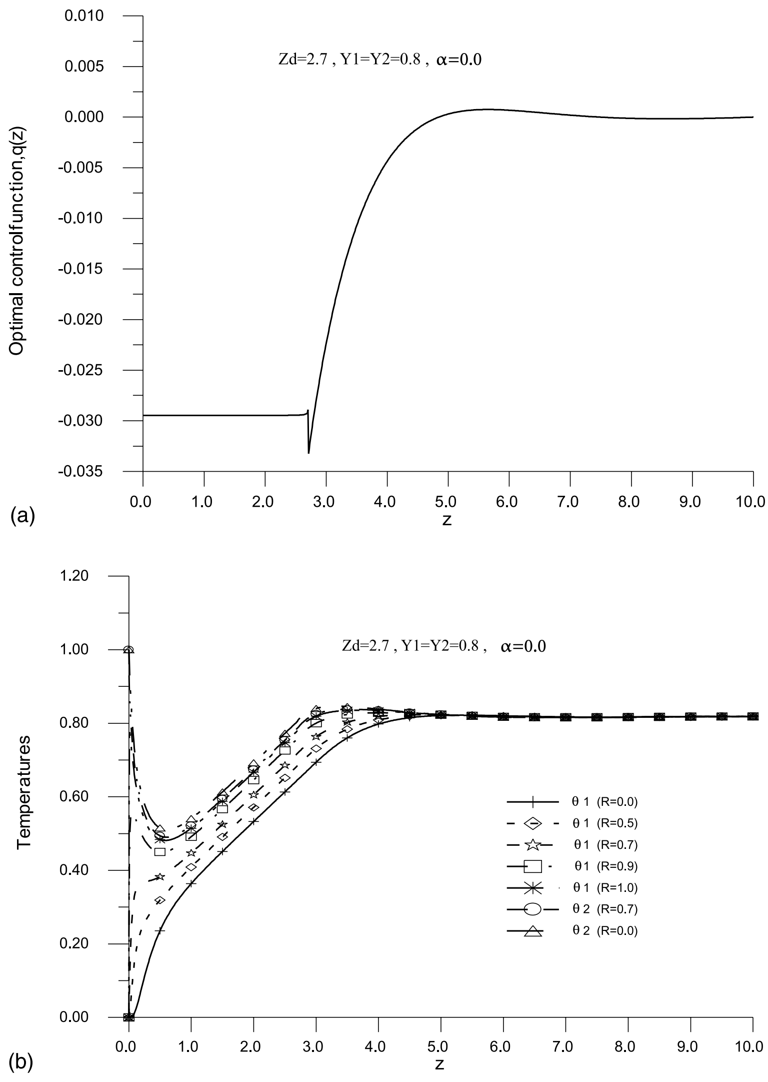


Fig. 4. (a) The estimated optimal control function $q(z)$ for $Y_1 = Y_2 = 0.8$, $z_d = 2.8$ and $\alpha = 0$ in test case 1. (b) The estimated temperatures, θ_1 and θ_2, for $Y_1 = Y_2 = 0.8$, $z_d = 2.8$ and $\alpha = 0$ in test case 1.

To illustrate the accuracy of the CGM in predicting $q(z)$ with optimal control analysis from the knowledge of desired thermal entry length and fluid temperature distributions, we consider following three specific examples.

One of the advantages of using the CGM is that the initial guesses of the unknown control function $q(z)$ can be chosen arbitrarily. In all the test cases considered here, the initial guesses of control function used to begin the iteration are taken as $q^0(z) = 0.0$.

We now present below the numerical experiments in determining $q(z)$ by the optimal control analysis:

8.1. Numerical test case 1

The optimal control problem is first examined by using the desired thermal entry length $z_d = 0.0$ and the desired fluid temperature distributions $Y_1(R,z) = Y_2(R,z) = 0.8$ for z greater than z_d. This requirement implies that the fluid temperatures are to be controlled as a uniform distribution started from the entrance by controlling the boundary heat flux. Due to the diffusion of heat, it is impossible to be achieved, however, the optimal thermal entry length (i.e. the shortest length for fluids to become desired uniform temperature) can thus be obtained.

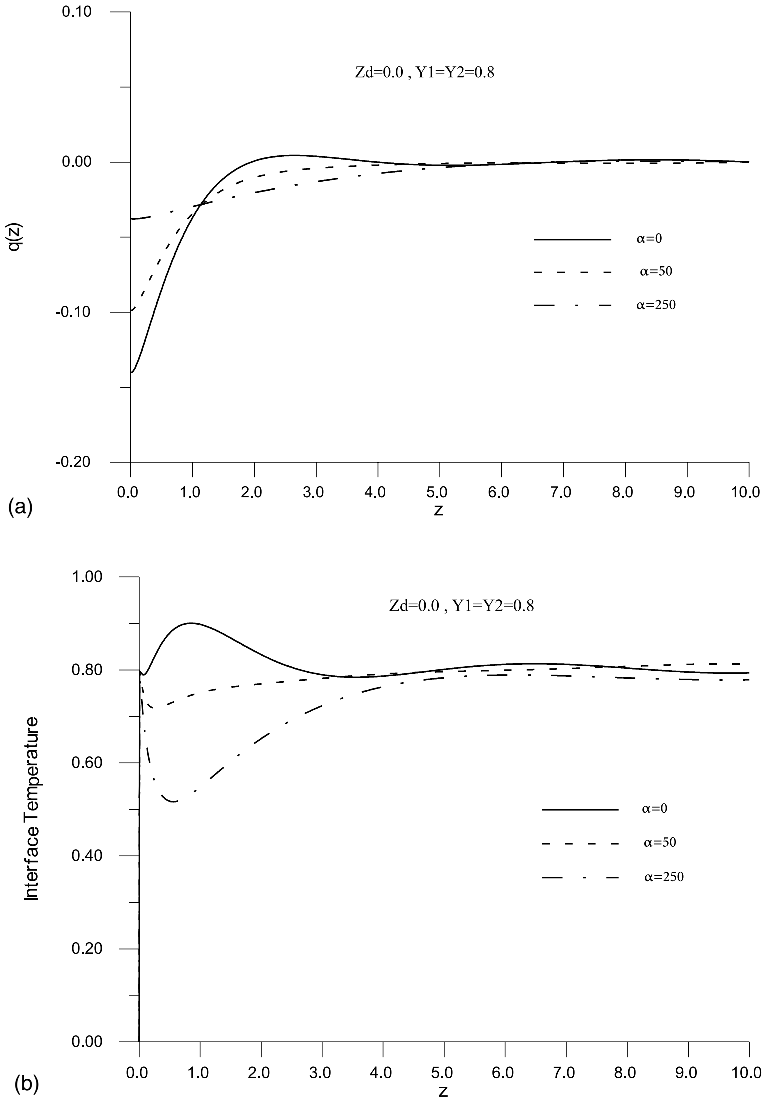


Fig. 5. (a) The estimated optimal control functions $q(z)$ for $Y_1 = Y_2 = 0.8$, $z_d = 0$ and $\alpha = 0$, 50 and 250 in test case 1. (b) The estimated temperatures at $R = 1.0$ for $Y_1 = Y_2 = 0.8$, $z_d = 0$ and $\alpha = 0$, 50 and 250 in test case 1.

For $\alpha = 0.0$, after 20 iterations (we have observed from the numerical experiments that 20 iterations are good enough to obtain the convergent solutions) the functional is calculated as $J = 0.169$ (CPU time at Pentium III-600 MHz PC is about 25 s) and the solutions for the optimal control function can be obtained. Fig. 2a shows the estimated control functions $q(z)$ and Fig. 2b illustrates the results for the estimated temperature distributions for two different fluids at some specified R along z. It is obvious from Fig. 2a and b that the fluid temperatures approached to $Y_1(R,z) = Y_2(R,z) = 0.8$ beyond $z = 2.0$. This implies that the optimal thermal entry length for the desired fluid temperatures $Y_1 = Y_2 = 0.8$ is about 2.0.

Once the fluid temperatures become uniform, the estimated control heat flux approaches to zero (i.e. insulated condition) beyond $z = 2.0$, this also implies that the insulation of the tube surface will maintain this temperature distribution, there is no need to add or to remove energy from the tube surface. The average error for the estimated temperatures is calculated as ERR = 0.96% for z from 2 to 10. The definition of average error ERR is given as

$$\mathrm{ERR}\% = \frac{\sum_{m=1}^{M}\left\{\sum_{n1=1}^{N1}\left|\frac{\theta_1(m,n1)-Y_1(m,n1)}{Y_1(m,n1)}\right| + \sum_{n2=1}^{N2}\left|\frac{\theta_2(m,n2)-Y_2(m,n2)}{Y_2(m,n2)}\right|\right\}}{[M(N1+N2)]} \times 100\% \tag{16}$$

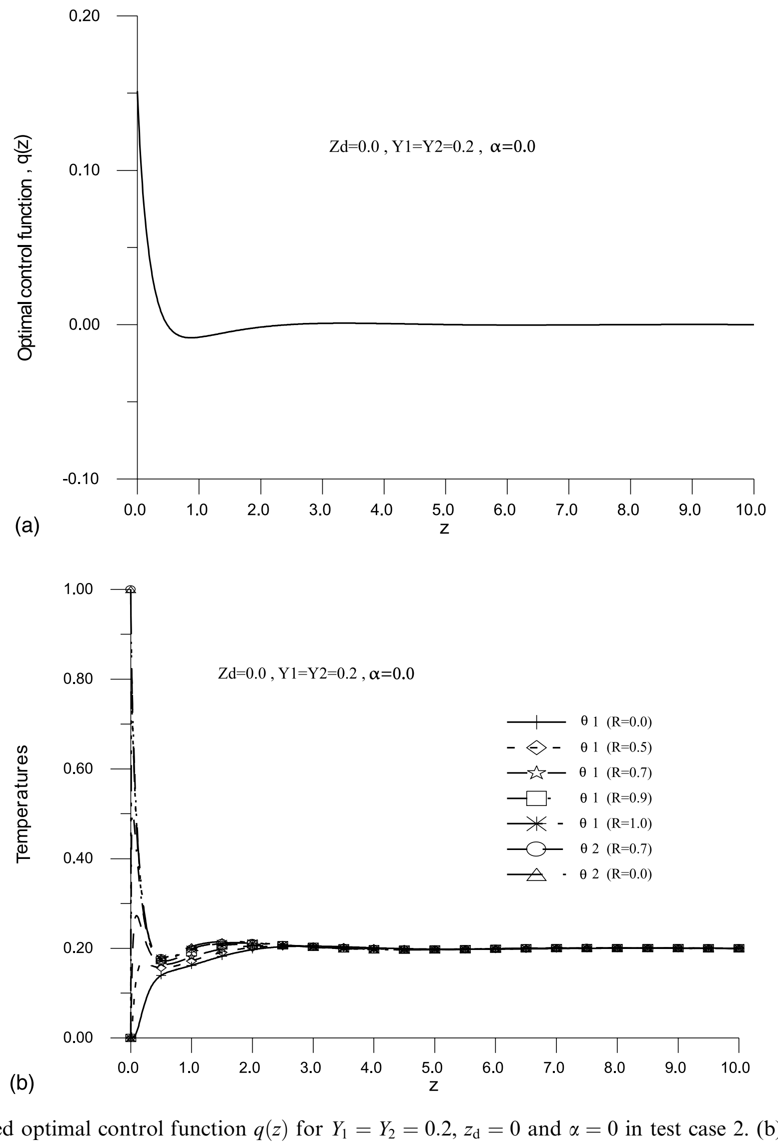


Fig. 6. (a) The estimated optimal control function $q(z)$ for $Y_1 = Y_2 = 0.2$, $z_d = 0$ and $\alpha = 0$ in test case 2. (b) The estimated temperatures, θ_1 and θ_2, for $Y_1 = Y_2 = 0.2$, $z_d = 0$ and $\alpha = 0$ in test case 2.

here m represents the index of discreted temperatures in z direction while $n1$ and $n2$ indicate the index of discreted temperatures in R direction. $(M \times N1)$ and $(M \times N2)$ represent the total number of discreted temperatures for fluid 1 and fluid 2 within specified domain.

When $z_d = 5.0$ is considered and all other conditions are the same as the previous case. After 20 iterations the functional is calculated as $J = 0.0065$ and the solutions for optimal control function and estimated temperatures are shown in Fig. 3a and b, respectively. The average error for the estimated temperature is calculated as ERR $= 0.73\%$ for z from 6.6 to 10.

It is obvious from Fig. 3a that the control function did not change until $z = 5.0$, however, due to the diffusion of heat, temperature distribution did not approach to 0.8 until $z = 7.0$. If one is asked preciously to have the thermal entry length equal to $z = 0.5$, the control function should be applied ahead of $z = 0.5$. For instant, if $z_d = 2.8$ is used, the estimated control function and temperatures are illustrated in Fig. 4a and b, respectively. From Fig. 4b we learn that the thermal entry length is now equal to 5.0.

In order to examine the effectiveness of the weighting coefficient α to the control function we consider the following numerical experiments: The numerical parameters are the same as the original conditions except that $\alpha = 50$ and 250 are

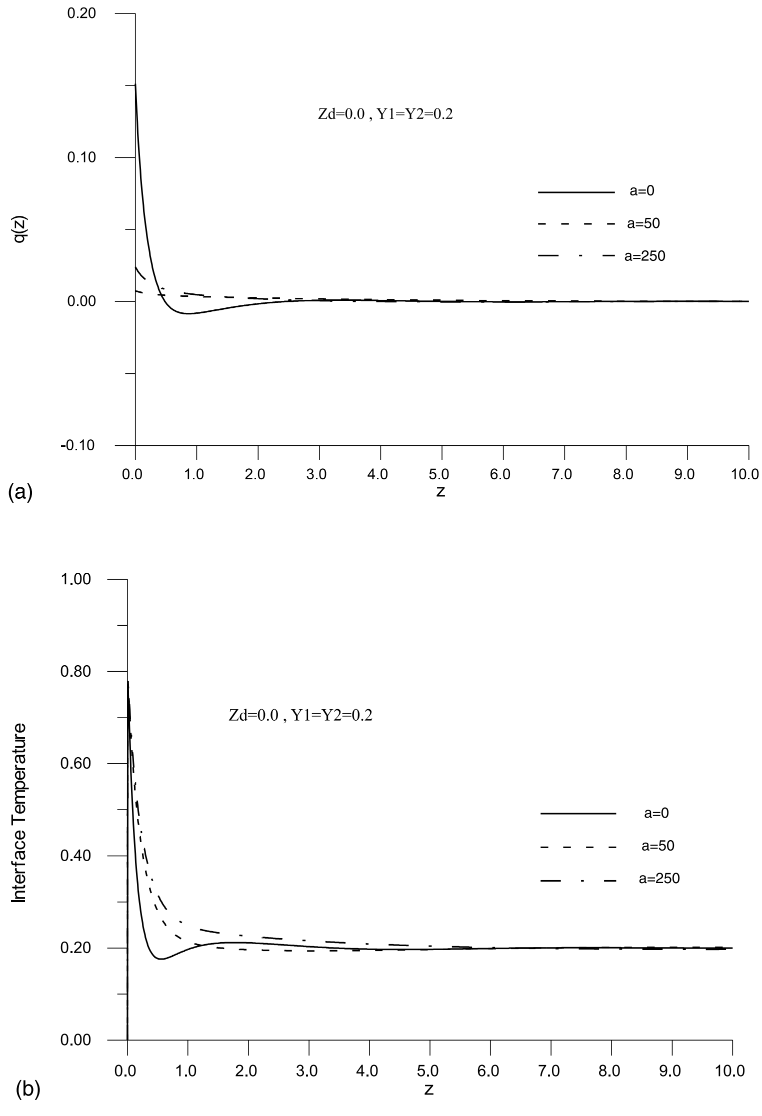


Fig. 7. (a) The estimated optimal control functions $q(z)$ for $Y_1 = Y_2 = 0.2$, $z_d = 0$ and $\alpha = 0$, 50 and 500 in test case 2. (b) The estimated temperatures at $R = 1.0$ for $Y_1 = Y_2 = 0.2$, $z_d = 0$ and $\alpha = 0$, 50 and 500 in test case 2.

used. After 20 iterations for each case, the optimal control function $q(z)$ can be determined. Fig. 5a shows the estimated $q(z)$ and Fig. 5b indicates the interface temperature distribution $\theta_1(1,z)$ and $\theta_2(1,z)$, (for the case $K_w = 0$, $\theta_1 = \theta_2$), for $\alpha = 0$, 50 and 250, respectively. It is clear from these figures that as the weighting coefficient increases, the absolute value of maximum strength of the control function and the accuracy of the estimated temperature are both decreased, and the optimal thermal entry length is increased. This implies that as $\alpha = 0$ we can always shorten the optimal thermal entry length and obtain more accurate temperature profile but at the same time higher strength of control function is needed.

If we integrate $q(z)$ with respect to z (i.e. calculate total supply energy Q), we found that Q is about equal to −0.1 for all three cases. This fact shows that as long as the temperature of fluids become uniform within the duct, the total energy should be the same for $\alpha = 0$, 50 and 250. The average errors ERR for the estimated temperature for $\alpha = 50$ and 250 are 0.75% and 2.08%, respectively for z from 5 to 10.

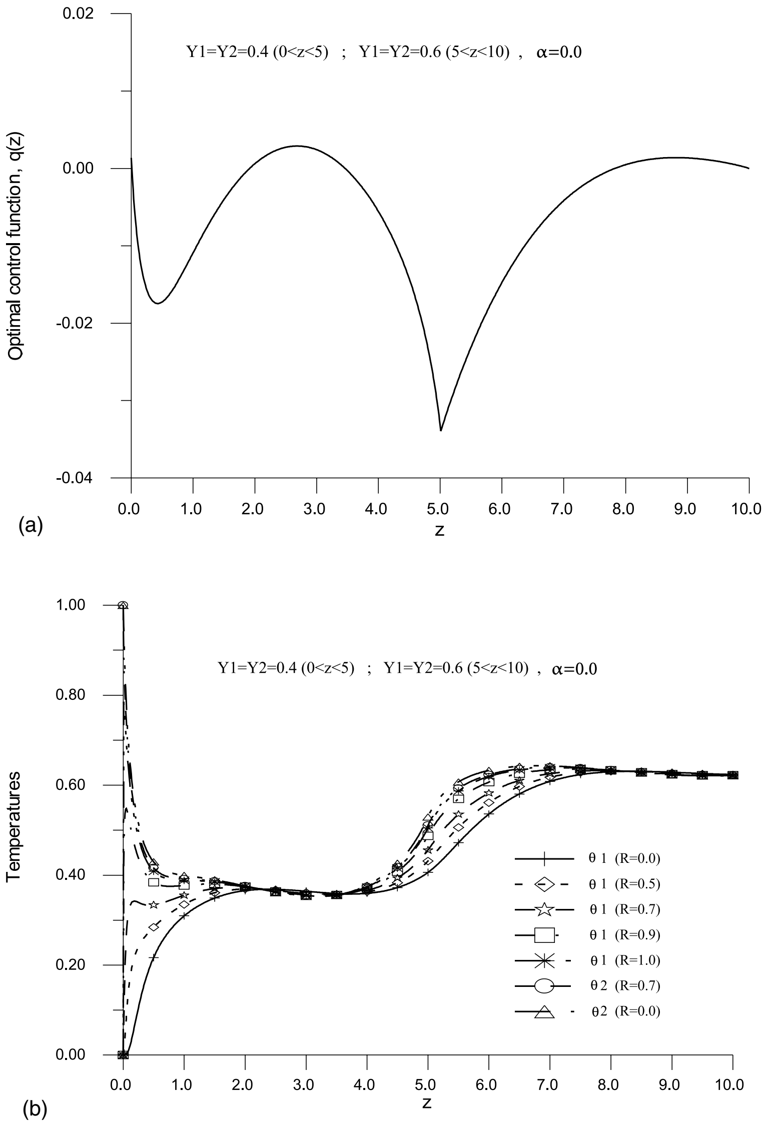


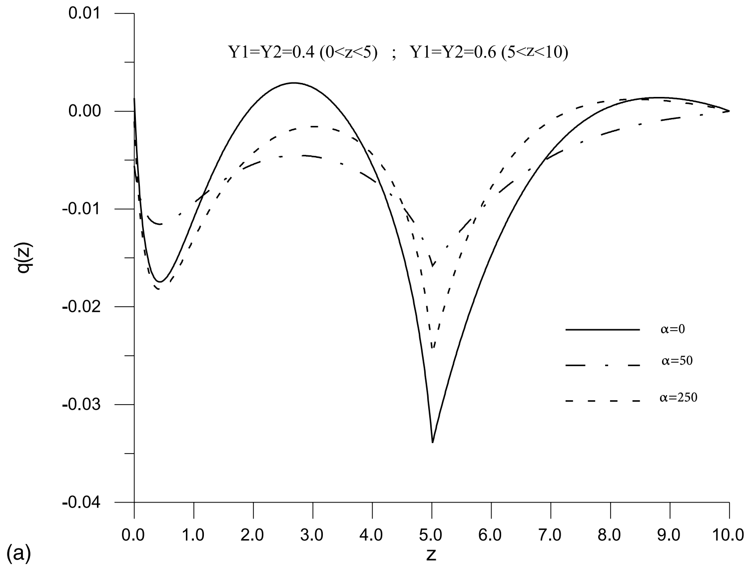
Fig. 8. (a) The estimated optimal control function $q(z)$ for $\alpha = 0$ in test case 3. (b) The estimated temperatures, θ_1 and θ_2, for $\alpha = 0$ in test case 3.

8.2. Numerical test case 2

In the second test case the numerical parameters are the same as were used in numerical test case 1 except that the desired fluid temperature distributions are now $Y_1(R,z) = Y_2(R,z) = 0.2$ for z greater than z_d, i.e. lower uniform fluid temperature is required.

The optimal control problem is first examined by using $\alpha = 0.0$. After 20 iterations (CPU time at Pentium III-600 MHz PC is about 25 s) the solutions for optimal control function $q(z)$ can be determined. Fig. 6a shows the estimated control function and Fig. 6b shows the results for the estimated temperature distributions. It is obvious from Fig. 6a and b that the estimated final temperature distribution can be satisfied with the desired temperature beyond $z = 2.5$. The average error for the estimated temperature is calculated as ERR = 0.69% for z from 2.5 to 10.

Next, when the weighting coefficients $\alpha = 50$ and 500 are used, after 20 iterations for each case, the optimal control function $q(z)$ can be determined. Fig. 7a and b show the estimated $q(z)$ and the interface temperature distribution for $\alpha = 0$, 50 and 500, respectively. Again as the weighting coefficient increases, the optimal thermal entry length also increases but the absolute value of maximum strength of the control function and the accuracy of the estimated



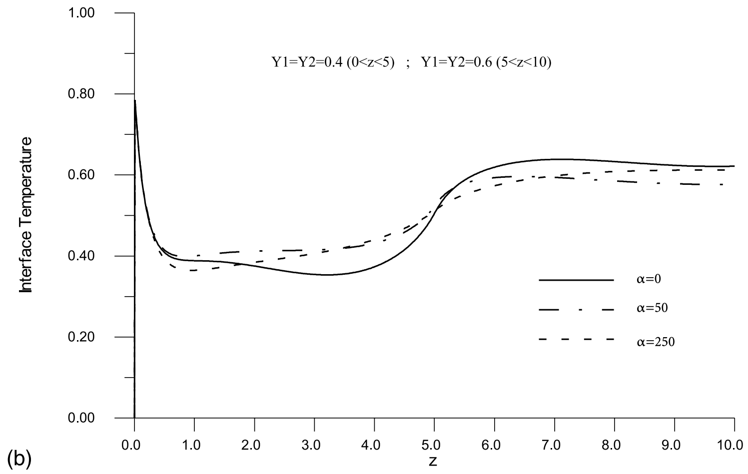


Fig. 9. (a) The estimated optimal control functions for $\alpha = 0$, 50 and 100 in test case 3. (b) The estimated temperatures at $R = 1.0$ for $\alpha = 0$, 50 and 100 in test case 3.

temperature decreases. The total removed energy is about $Q = 0.014$ for all three cases. The average errors ERR for the estimated temperature for $\alpha = 50$ and 500 are 2.84% and 1.27%, respectively for z from 2.5 to 10.

8.3. Numerical test case 3

A stricter situation is examined in the third test case. The desired temperature distributions for two fluids are in the following forms

$$\begin{cases} Y_1(R,z) = Y_2(R,z) = 0.4, & 0 \leqslant z \leqslant 5.0 \\ Y_1(R,z) = Y_2(R,z) = 0.6, & 5.0 < z \leqslant 10.0 \end{cases} \tag{17}$$

which represents a step change of temperature profile in the duct.

The optimal control problem is performed firstly by using $\alpha = 0.0$. After 20 iterations the solutions for optimal control function $q(z)$ can be determined. Fig. 8a shows the estimated control function $q(z)$ while Fig. 8b shows the result for the estimated temperature distributions. From those figures we learned that except for some delay due to diffusion of heat, the requirement of temperature distribution can be satisfied.

Then $\alpha = 50$ and 100 are considered, after 20 iterations the solutions are shown in Fig. 9a and b for the control function $q(z)$ and interface temperature $\theta_1(1,z)$ and $\theta_2(1,z)$, respectively. The similar phenomena that were discussed previously can also be observed here.

From above three numerical test cases we concluded that the CGM can be applied successfully in this coupled optimal control problem for predicting the boundary control function $q(z)$.

9. Conclusions

An optimal control algorithm based on the CGM with adjoint equation was successfully applied for the solution of entrance concurrent flow problem in determining the optimal control function. Three test cases involving different desired fluid temperature distribution and thermal entry lengths were considered. The results show that the CGM does not require a priori information for the functional form of the unknown control functions and the optimal solutions can be obtained within a very short computer time.

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References

[1] M.D. Mikhailov, B.K. Shishedjiev, Coupled at boundary mass or heat transfer in entrance concurrent flow, Int. J. Heat Mass Transfer 19 (1976) 553–557.
[2] R.P. Stein, The graetz problem in concurrent flow double pipe heat exchangers, Chem. Eng. Prog. Symp. Ser. 59 (1965) 76–87.
[3] M.D. Mikhailov, General solutions of the diffusion equations coupled at boundary conditions, Int. J. Heat Mass Transfer 16 (1973) 2155–2164.
[4] A.G. Butkovskii, A.Y. Lerner, The optimal control systems with distributed parameters, Automat. Remote Control 21 (1960) 472–477.
[5] R.A. Meric, Finite element and conjugate gradient methods for a nonlinear optimal heat transfer control problem, Int. J. Numer. Meth. Eng. 14 (1979) 1851–1863.
[6] R.A. Meric, Finite element analysis of optimal heating of a slab with temperature dependent thermal conductivity, Int. J. Heat Mass Transfer 22 (1979) 1347–1353.
[7] C.J. Chen, M.N. Ozisik, Optimal heating of a slab with a plane heat source of timewise varying strength, Numer. Heat Transfer, Part A 21 (1992) 351–361.
[8] C.J. Chen, M.N. Ozisik, Optimal heating of a slab with two plan heat sources of timewise varying strength, J. Franklin Inst. 329 (1992) 195–206.
[9] C.J. Chen, M.N. Ozisik, Optimal convective heating of a hollow cylinder with temperature dependent thermal conductivity, Appl. Sci. Res. 52 (1994) 67–79.

[10] C.H. Huang, A non-linear optimal control problem in determining the strength of the optimal boundary heat fluxes, Numer. Heat Transfer, Part B 40 (2001) 411–429.
[11] C.H. Huang, An optimal control problem in estimating the optimal control force for the force vibration system, Int. J. Numer. Meth. Eng. 52 (2001) 1323–1335.
[12] O.M. Alifanov, Inverse Heat Transfer Problem, Springer-Verlag, Berlin, Heidelberg, 1994.